SHORT CIRCUIT

Canberra Mathematical Association Inc.

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CMA MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website:
http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

NEWSLETTER

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member but we do encourage you to join.

Short Circuit welcomes all readers.

NEWS AND COMMENT

It has come to our attention that the year number 2025 is the square of the number 45.

As such, it is the sum of the first 45 odd positive integers: 1 + 3 + … + 89 .

The number 45 is itself the sum of the first 9 positive integers, and then the formula for the first *n* consecutive cubes shows that 2025 is the sum of the first 9 cubes: 1 + 8 + … + 729.

Remarkably, the tangent of the angle 2025° is 1. That trigonometric curiosity happened previously in 1845 and will not happen again until the year 2205.

Clearly, January is a slow news month for Short Circuit. May it be for all teachers of mathematics a stress-free time in preparation for a fruitful new year.

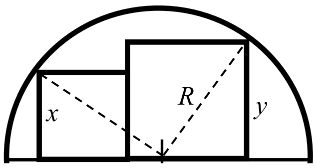
CANBERRA MATHEMATICAL ASSOCIATION

Inside:

PUZZLES

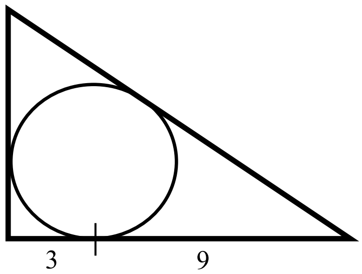
1. Surprising squares

Two squares are constructed side-by-side on the diameter of a semicircle with each square just touching the circumference.



In terms of the radius R and the sides of the two squares x and y, find an expression for the combined areas of the squares.

2. Incircle



The circle inscribed in a right-angled triangle has the distances shown in the diagram.

i What is the area of the triangle?

ii What geometrical facts were needed to solve this problem?

iii If instead of lengths 3 and 9 we had lengths u and v, is there a general expression for the area of the triangle?

3. Alphametic

This sum is correct linguistically and also arithmetically when each letter is replaced by a different particular digit from 0 to 9.

```
   H A L F
 F I F T H
 T E N T H
 T E N T H
 T E N T H
 W H O L E
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i How many one-to-one mappings from the ten letters to the ten digits are there?

ii Find one that works.

PUZZLE SOLUTIONS from Vol 15 No 12

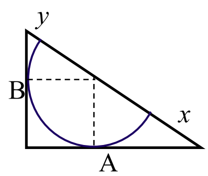
1 Fun calculation

To the nearest second, how much short of a week is $100(e/\varphi)$ hours?

A week is 604800 seconds. A calculator gives $100\ e/\varphi \approx 167.9990561$ in hours. So, this is about 604796.6 seconds which is a little over 3 seconds less than the expression.

2 Bits of hypotenuse

A semicircle is drawn inside a right-angled triangle as shown. Find the lengths of the segments x and y.



What if the triangle has sides A, B, $\sqrt{(A^2 + B^2)}$?

Using the three similar triangles we have

$(x + r)/r = \sqrt{(A^2 + B^2)}/B$ (1)

and $(y + r)/r = \sqrt{(A^2 + B^2)}/A$ (2)

where r is the radius of the semicircle.

The hypotenuse is $\sqrt{(A^2 + B^2)} = x + y + 2r$.

Adding equations (1) and (2) gives $(x + y + 2r)/r = \sqrt{(A^2 + B^2)}/(1/A + 1/B)$. So that, $1/r = 1/A + 1/B$. (A nice result!)

That is, $r = AB/(A + B)$. This value for r can be used in equations (1) and (2) to obtain values for x and y.

In the case B = 3 and A = 4 we find $x = 8/7$ and $y = 3/7$. The radius is 12/7 and so, the hypotenuse has length 35/7 as expected.

ABOUT THE CMA

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC. INC

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We're on the Web!
http://www.canberramaths.org.au/

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.

It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

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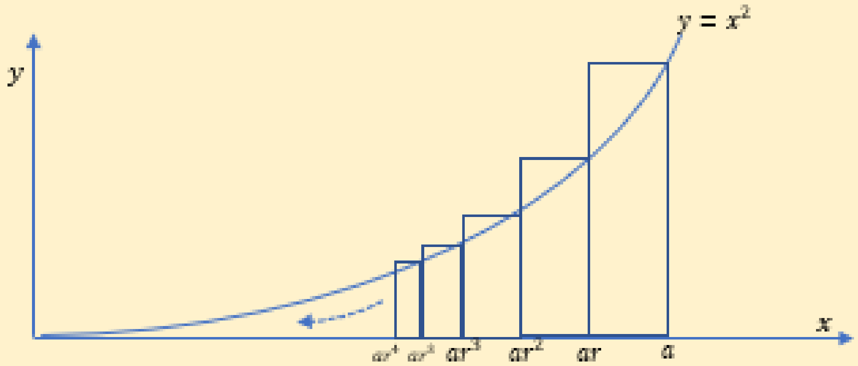
In 1666, Isaac Newton (1643-1727) calculated areas by antidifferentiation, and this work contains the first clear statement of the Fundamental Theorem of the Calculus. Perhaps less well known is a result made by Pierre Fermat (1601-1665) as early as 1629 that uses geometric sums in a novel way to find the area under the curve $y = x^2$. The solution would make an ideal introduction to the first steps in integral calculus.

It starts with a graph of the function between $x = 0$ and $x = a$ and a few of the circumscribed rectangles, but in Fermat's solution the width of each rectangle, constructed from the right, diminishes so that the left hand edges of each rectangle line up with x values that form a geometric progression whose first term is a and common ratio is r where $r < 1$.



Even though the rectangles continue indefinitely toward the y axis we can sum their areas for we know their widths, given as $(a - ar), (ar - ar^2), (ar^2 - ar^3), \ldots$ and their heights, given as $f(a) = a^2, f(ar) = a^2r^2, f(ar^2) = a^2r^4, \ldots$

Thus, after taking a^3 out as a common factor, the sum S can be written:

$$S = a^3[(1 - r)(1 + r^3 + r^6 + \cdots)]$$

The summation in the second bracket is infinitely geometric with first term 1 and common ratio $r^3 < 1$ and so has the limiting sum $\frac{1}{1-r^3} = \frac{1}{(1-r)(r^2+r+1)}$. When this expression is substituted in, the sum becomes

$$S = \frac{a^3}{(r^2 + r + 1)}$$

At this point, Fermat's genius kicks in. If r is allowed to approach 1 the widths of all the rectangles reduce, and the parts of the rectangles above the curve reduced also. It is not difficult to imagine that the sum of the areas of all the rectangles approaches the area under the curve.

This implies that $r^2 + r + 1 \to 3$ and $S \to \frac{a^3}{3}$.