SHORT CIRCUIT

Canberra Mathematical Association Inc.

VOLUME 15 NUMBER 6 JUNE 2024

NEWS AND COMMENT

One of the aims of the CMA (listed on page four of this newsletter) is 'the promotion of mathematical education to government through lobbying'. Lately, there have been opportunities to do just that.

Beginning on page three you will find the response CMA made to the recent Inquiry by the ACT government into numeracy and literacy education. We raised some issues and now look forward to a response.

A related matter, again calling on CMA's advocacy role, is the education system's continuing failure to meet the needs of some students in mathematics. In particular, as outlined in the May edition of Short Circuit, a delegation from CMA attended a summit concerning the mathematical education of Aboriginal and Torres Strait Islander students.

This was organised by ATSIMA and AAMT. A commitment statement was produced and action plans are being drafted.

For a start, AAMT has written to the Education ministers in every jurisdiction reminding them of the situation, apprising them of ATSIMA's initiative, and requesting an opportunity to discuss the matter at an Education Ministers Meeting.

AAMT CEO Allan Dougan has suggested that CMA members, along with those from the other affiliates, might help this process along by writing to their education ministers urging them to pay attention.

Some thoughts arising from the ATSIMA summit are on the following page.

For those looking for a motivational activity for their students, consider the Canberra Mathematics Talent Quest 2024. Details are on page 2.

MEMBERSHIP

Memberships run from 1 Jan to 31 Dec. each year. Membership forms may be downloaded from the CMA website:
http://www.canberramaths.org.au

The several benefits of Membership of CMA may be found on the website.

NEWSLETTER

The CMA newsletter, Short Circuit, is distributed monthly to everyone on our mailing list, free of charge and regardless of membership status.

That you are receiving Short Circuit does not imply that you are a current CMA member but we do encourage you to join.

Short Circuit welcomes all readers.

Inside:

CANBERRA MATHEMATICAL ASSOCIATION

ATSIMA SUMMIT

Following from the April summit meeting attended by representatives from every state and territory professional association of mathematics teachers, a CMA subcommittee is now seeking to translate into action the insights and resolutions from the summit.

The meeting was an initiative of the Aboriginal and Torres Strait Islander Mathematics Alliance under the leadership of Professor Chris Matthews together with the AAMT. The Commitment Statement produced by the summit can be seen in the May edition of this newsletter. At the head of the list of commitments was the idea of 'truth telling'.

In the belief that education is likely to be more effective (whoever the students) when the teacher has some understanding of and empathy for those they are attempting to teach, this newsletter will from time to time suggest articles, books, and videos that readers are encouraged to peruse.

There is, for example, a fine documentary The Last Daughter, available on Netflix, which is Wiradjuri woman Brenda Matthews's story about the consequences of being a member of the stolen generations. Brenda Matthews is the author of the book with the same title.

Another informative read is Growing up Aboriginal in Australia edited by Anita Heiss. The book contains short chapters about their childhoods by fifty-two aboriginal contributors. This link goes to a YouTube clip about the book.

CMTQ 2024

THE CANBERRA MATHEMATICS TALENT QUEST 2024

The National Mathematics Talent Quest has provided a venue to showcase the creative thinking skills of students in Australia for many years. To be eligible to enter the national quest a project has to be successful in a similar quest at the state level. Students throughout the ACT put considerable time and effort into mathematics assignments and projects and now have a means to get local or even national recognition and encouragement for their work.

Students may participate in the quest in one of three categories:

- Submit an **individual** entry
- Be part of a **small group** (up to 6 students)
- Be part of a **whole class** entry (7 or more students)

Entry is free.

All students from Kindergarten to Year 12 in the ACT are eligible to submit an entry.

The project or assignment can be the student's own idea or a teacher's set task with an outstanding student response.

The projects or assignments may be presented in any format including:

- Essays, scripts, stories, poems, diaries, illustrated texts, newspaper format or any other form of writing
- Posters
- Videos
- Models – static or working
- Computer based (coding)
- PowerPoint presentation
- Spreadsheet or database.

We will give you information about submission of entries in July. Due to COVID, entries in most of the states have moved to digital submissions, usually using PowerPoint. Entries that win their category are automatically entered in the national competition. Entries in the National Mathematics Talent Quest must be submitted electronically. This is due to the way the entries are evaluated by judges in all the states and territories. Schools can submit up to two entries per category (individual, group or class) per year group to be assessed by an ACT judging panel. Follow the links to see some examples of student work from Victoria, NSW and WA, and check the *handbook*.

You can start anytime but the entry date is Wednesday 14th August 2024.

Updates will be provided in *Short Circuit* and on the CMA webpage: http://www.canberramaths.org.au/

RESPONSE TO GOVT. INQUIRY

Literacy and numeracy in ACT public schools.

A response from the Canberra Mathematical Association to the recent ACT Government Inquiry

Introduction

The Canberra Mathematical Association represents teachers of mathematics in the ACT across all jurisdictions and organisational levels. As a professional association, our focus is specifically on mathematics education. From this perspective, we would like to share our response to the review of literacy and numeracy in ACT public schools. We have tried to ensure that our suggestions are all practical and will have a positive impact on the lives and careers of all teachers of mathematics around the ACT.

Suggestion: to engage all teachers of mathematics in a community of learners

The Canberra Mathematical Association provides a framework for mathematics teachers to work collaboratively within a supportive community. AITSL standards (Focus area 7.4: Engage with professional teaching networks and broader communities) acknowledge the importance of teachers engaging with professional organisations in developing their own understanding of their curriculum area and professional practice.

Members of the Canberra Mathematical Association have a wealth of experience and knowledge in mathematics education. This experience extends from early childhood learning all the way to tertiary education. These members are passionate about mathematics education and are keen to share their expertise and knowledge with other teachers in the ACT. This organisation is a significant resource in the campaign for improving numeracy standards.

Teachers within the ACT are required to be registered annually with the Teacher Quality Institute. This registration process aims to ensure higher professional standards. To bring together the TQI registration requirements and the function of the CMA to support a community of mathematics teachers, we would propose:

Membership of a professional association such as CMA would give a teacher the equivalent of a 2 hours TQI accredited program.

That the ACT government should require all secondary schools and colleges to be institutional members of the Canberra Mathematical Association.

Primary schools and early learning schools should also be encouraged to take up institutional membership.

Teachers who take up individual membership of the Canberra Mathematical Association should receive financial reimbursement from the ACT government.

Suggestion: to support early career teachers

Early career teachers play a significant role in the education system. Research has suggested that it is critical to engage young teachers within the first five years of their career and to nurture their talents and enthusiasm appropriately. Structures need to be defined that will provide this support to young teachers so that we can look forward to their continued contribution through a lifetime career in education.

Sadly, many young teachers do not survive the first five years of their career. Many fully trained teachers seek employment in other industries within the first few years of their teaching career. To mitigate against this, we would propose:

The development of an early career teacher network within the ACT. The Canberra Mathematical Association would be very interested in supporting such an initiative in relation to helping teachers of mathematics in the first five years of their teaching career. The experience and expertise of our members could provide strong guidance and direction for young teachers. Such an initiative could become an informal network of colleagues working together. A small amount of financial support to implement this plan may be required. Release time for early career teachers to attend mentoring at network meetings may also be required.

Teachers learn much from seeing other teachers in action. It would be good to develop a system where early career teachers can regularly observe more experienced teachers in the classroom. This may already be a part of the informal organisation within some schools. However, a more formal approach to lesson observation would have significant benefits for early career teachers.

International study tours, where teachers get to visit schools in other places, can provide inspiration and motivation to young teachers to achieve their potential. The Australian Association of

NEWSLETTER OF THE CANBERRA MATHEMATICAL ASSOCIATION INC. INC

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ABOUT THE CMA

The Canberra Mathematical Association (Inc.) is the representative body of professional educators of mathematics in Canberra, Australia.
It was established by, among others, the late Professor Bernhard Neumann in 1963. It continues to run - as it began - purely on a volunteer basis.

Its aims include

* the promotion of mathematical education to government through lobbying,
* the development, application and dissemination of mathematical knowledge within Canberra through in-service opportunities, and
* facilitating effective cooperation and collaboration between mathematics teachers and their colleagues in Canberra.

THE 2024 CMA COMMITTEE

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Theresa Shellshear is CMA's COACTEA representative.

Bruce Ferrington is CMA's AAMT representative.

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http://www.facebook.com/pages/Canberra-Mathematical-Association/110629419011275

SPHERICAL THINKING

In the early years of high school, students are shown a cunning way of finding the flat area enclosed by a circle. The idea is to dissect a circular disc along radial lines, into a great many equal sized sectors. These are rearranged into a shape approximating a parallelogram and the area is then announced to be its length times its width.

To be fully convinced that this result is correct, there are a few things a student must first be willing to believe. These include that the separate pieces in a dissection have areas that add to the area of the whole shape; that the circumference of a circle has a definite and known relationship to its radius; and that the bumpy edges of the approximate parallelogram can be made as close as we like to the straight edges of a genuine parallelogram by cutting the disc into ever smaller sectors.

As a young student I accepted the result without question although I now see that there was room for doubt. (Is doubt permitted in the maths classroom?) Later, I was asked to believe the well-known formulae for the volume and surface area of a sphere. There was a 'proof' involving flattened out pieces of orange peel but ultimately these had to be taken on faith pending one's initiation into something called calculus.

Still more mysterious, even for fully fledged mathematics teachers, is the idea that there might be a sphere in four or more dimensions for which a volume and a surface area could be meaningfully calculated. (Such formulae exist and can easily be looked up on the internet.) Diagrams, however, are impossible for those of us living in a three-dimensional world.

High school mathematics can be an introduction to procedures and formulae about circles, spheres and the like that may occasionally be useful in practical contexts. However, another educational outcome, with implications beyond computation, might be the enhancement of a student's inclination and ability to reason, and not just mathematically.

It seems that a starting point towards understanding the properties of spheres will have to be good definitions of the elements fundamental to the problem.

A discussion about spheres might go something like the following, not because anyone desperately needs to know how to calculate the volume of an *n*-dimensional sphere, but to illustrate mathematical thinking as an exploration of the otherwise unimaginable.

The word *vector* refers to symbols $(a_1, a_2, a_3, \ldots, a_n)$ For this discussion, the slots $a_1, a_2, a_3, \ldots, a_n$ in the vector will be filled with real numbers and we should bear in mind that the argument might fail if some slots are populated by other types of numbers or by other objects. We assume that the usual distance formula derived from coordinate geometry has the right properties when applied to vectors of real numbers (and postpone this part of the discussion for another day).

Consider subsets of vectors that have a fixed distance from the zero vector. Call them *n*-spheres according to the number *n* of slots in the vectors. The 2-spheres are identified with circles, and we have vectors (a, b) with $\sqrt{a^2 + b^2} = r$ a constant. The vectors (a, b) with $\sqrt{a^2 + b^2} < r$ correspond to a circular disc bounded by the circle.

Suppose your students are convinced that in circles of any size the circumference C is always a fixed multiple of the radius r. We might write $C = kr$ without knowing much about k.

We wish, if possible, to assign a number depending only on r that measures the aggregate of vectors (a, b) with $\sqrt{a^2 + b^2} < r$. (This corresponds to the intuitive idea of *area* in physical space.) To this end note that if the radius increases by a small amount δr, then the circumference grows to make a band that includes all vectors (a, b) with $r \leq \sqrt{a^2 + b^2} < r + \delta r$. We can define the extra area so obtained to be $kr\delta r$. Thus, the rate of change of the area at radius r is just kr.

By allowing the radius of circles centred at $(0,0)$ to grow from 0 to r in tiny steps of δr, every vector in

the disc is included. Following Newton and Leibniz, we can assign a number to the whole collection of circular bands by evaluating the integral $\int_0^r kt\ dt$. Thus, we obtain the area $\frac{k}{2}r^2$, and after determining that $k = 2\pi$, we have the usual area expression for circles, πr^2.

Vectors of three real numbers (a, b, c) with the requirement $\sqrt{a^2 + b^2 + c^2} = r$, correspond to surfaces of physical spheres. If a is fixed at some value A, we have equivalently, $\sqrt{b^2 + c^2} = \sqrt{r^2 - A^2}$, which makes the set of vectors (b, c) a circle with radius $\sqrt{r^2 - A^2}$. According to the previous result, its area is $\pi(r^2 - A^2)$

As before, we wish to assign a number depending on r (call it the volume) that measures the aggregate of vectors with $\sqrt{a^2 + b^2 + c^2} < r$. If the number a changes by a small amount δa, we can say that the volume changes by $\pi(r^2 - a^2)\delta a$. In effect, the sphere has been dissected into thin circular pieces with thickness δa. Again, calling on calculus, we evaluate an integral $\pi \int_{-r}^{r}(r^2 - t^2)\ dt$ to obtain the familiar expression $\frac{4}{3}\pi r^3$.

Alternatively, the sphere can be dissected into concentric thin spherical shells. A typical shell has surface area s and volume $s\delta r$ so that the rate of change of volume at radius r is just s. Hence, by differentiation, the surface area of a sphere when the volume is $\frac{4}{3}\pi r^3$ must be $4\pi r^2$.

Similarly, we can consider vectors (a, b, c, d) constrained by $\sqrt{a^2 + b^2 + c^2 + d^2} \leq r$. When $a = A$ is fixed while b, c and d vary, we have a subset of the vectors such that $\sqrt{b^2 + c^2 + d^2} \leq \sqrt{r^2 - A^2}$. This is a ball (b, c, d) with radius $\sqrt{r^2 - A^2}$. According to the previous result, it has volume $\frac{4}{3}\pi(r^2 - A^2)^{\frac{3}{2}}$. The ball is a subset of the vectors (a, b, c, d) and we can define the measure of the union of all such subsets, with a varying from $-r$ to r, by the integral $\frac{4}{3}\pi \int_{-r}^{r}(r^2 - t^2)^{\frac{3}{2}}dt$. This evaluates to $\frac{1}{2}\pi^2 r^4$, which we might call the volume of a 4-dimensional hypersphere.

This process can be iterated indefinitely to obtain the volumes of hyperspheres in any number of dimensions.

The meaning of a sphere in some given number of dimensions depends on what quantities go into the vector slots. Typically, the slots contain physical distance measurements, but they could contain anything that is described by a real number.

It must be admitted that there remains room for doubt arising from the notation, from unclear definitions, and from gaps in the argument. Complete rigour seems always to be somewhere further down the track.

PT

FROM PAGE THREE

Mathematics Teachers, of which the Canberra Mathematical Association is an affiliated member, organises such tours. The ACT government could provide incentives for early career teachers to attend such study tours.

Suggestion: to develop the professional knowledge of all teachers of mathematics

Mathematics is a subject area that requires specific and in-depth knowledge. Sadly, there is sometimes a criticism that teachers responsible for this subject are not familiar or confident with the mathematical content that they teach. This is compounded by the fact that there are many teachers responsible for teaching mathematics in secondary schools who are teaching "out of field", that is, they are trained to teach subjects other than mathematics.

Part of the work of the Canberra mathematical association is to run workshops, seminars, and conferences. Through these activities, we utilise the skills of exceptional mathematicians and educators who share their knowledge to help others become better teachers of mathematics. Through our work as a professional association, we have regularly engaged with national and international guest speakers who have graciously shared their time and knowledge with us in many ways.

It Is important that any review of numeracy across the ACT should consider ways that will improve the professional knowledge of all teachers of mathematics. We propose:

That the ACT government subsidises attendance for teachers of mathematics at the Canberra Mathematical Association annual conference

That the ACT government supports a regular pro-

gramme of workshops and seminars run by the Canberra Mathematical Association for teachers of mathematics across the ACT

That teachers of mathematics who do not have qualifications in this subject should be encouraged and supported to seek further education at a tertiary level in this area.

That a support network for "out of field" teachers of mathematics should be established by the ACT government with the support of the Canberra Mathematical Association.

Suggestion: to engage girls in mathematics education

There is a reported disparity in gender engagement in mathematics. Our education system should not accept this. We should be doing everything possible to encourage and support female students in mathematics and they should be given every opportunity to succeed.

This issue is part of a bigger problem related to low participation rates for females in Science, Technology, Engineering and Maths (STEM) subjects. This low participation rate for females in all aspects of STEM subjects is commonly discussed in the media and recorded in academic research. A systemic solution is required to address this problem and it needs to be a high priority on the education agenda.

This review of literacy and numeracy needs to address female participation in mathematics. We propose:

Providing opportunities for girls to meet and learn from successful females who work in STEM related industries. Workshops and presentations could be organised to bring female students together to hear the stories of other females who have pursued careers in STEM subjects. From the example of others, some female students may be inspired to persist with mathematics in high school and college.

The Australian Mathematical Sciences Institute (AMSI) has this issue as a policy priority, identifying equity and diversity as its priority point. AMSI has developed significant resources related to girls and maths. The ACT government could make formal connections with the AMSI team to provide advice and guidance in how to engage girls in maths.

PUZZLE SOLUTIONS from Vol 15 No 5

1. Days and centuries

No century begins with a Tuesday, Thursday or Sunday. Can you prove it?

The proof depends on the length of the repeating cycle of years and leap years. Year numbers divisible by 4 are leap years except when they are divisible by 400. A period of 400 years has a number of days that is divisible by 7. So the cycle repeats at intervals of 4 centuries. Therefore, three days of the week must be missed.

2. Missing magic

Find the missing numbers to complete this magic square. Is your solution unique?

4	9	8
11	7	3
6	5	10

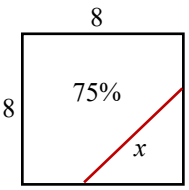
Unique. If say, 10 is changed by +/– 1, so are 9, 8 and 11. Then 3 would have to change by –/+ 2 and the middle row would then be incorrect.

3. Fish

One morning a fisherman catches 50 fish from a lake. He puts small tags on them and returns them to the lake. In the afternoon he catches 40 fish and 10 of them are tagged. Estimate the number of fish living in the lake.

The proportion of tagged fish in the second catch approximates the proportion in the lake. So, there are about 200 fish.

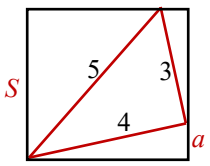
4. Cutting corners



The triangle has area 16 and sides $4\sqrt{2}$. So, $x = 8$.

5. Area

A square encloses a 3-4-5 triangle as in the following diagram. What is the area of the square?



By similar triangles, $(S - a)/3 = S/4$. By Pythagoras, $S^2 + a^2 = 16$. So, $a = S/4$ and after substitution for a, we find $S^2 = 256/17$.